Linear Programming Simplex Method

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FOR B.COM(H)
SEMESTER IV, SECTIONS A & B

Introduction to Simplex Method

- In Graphical method, we used only two variables, x & y to plot on the graph
- Beyond 2 variables, graphical method becomes difficult to solve
- In reality, Linear Programming Problems do not have only 2 variables with pure inequalities; there could be multiple variables with mixed constraints
- Simplex method allows mathematical solutions to linear programming problems

Terms you should know

1. Slack Variable

- To convert an inequality to an equality, we *add* a variable to the left side of a *less than or equal to* constraint
- It makes up for the slack/deficiency on the left side
- For Instance, X + 3Y <=20

To convert this inequality into an equality, we add a slack variable, "*s*" on the L.H.S.

The equation now becomes X + 3Y + s = 20

2. Surplus Variable

- Similar to slack variable, to convert an inequality to equality, we *subtract* a variable from the left side of a *greater than or equal to* constraint
- This is done to reduce/remove the excess on the left side
- For instance, $X + 3Y >= 20$

To convert this inequality into an equality, we subtract a surplus variable "*s*" from the L.H.S.

The equation now becomes $X + 3Y - s = 20$

How to Solve an LPP using Simplex

MAXIMIZATION CASE

- Refer to Scanned notes uploaded online
- Refer to videos sent for detailed explanation
- Refer to next slide for **Example 3, Page 4.8**

Step 1 – Problem Formulation

Let the number of chairs produced be x
Let the number of tables produced be y

Profit is Rs.20/chair and Rs.30/table

Based on the information and constraints given, the LPP can be formulated as follows:

Maximize $Z = 20x + 30y$

Subject to:

$3x + 3y <= 36$

$5x + 2y <= 50$

$2x + 6y <= 60$

$x, y >= 0$

Step 2 - Introduction of Slack/Surplus Variables, as per question

Maximize $Z = 20x + 30y + 0s1 + 0s2 + 0s3$

Subject to:

$3x + 3y + s1 = 36$

$5x + 2y + s2 = 50$

$2x + 6y + s3 = 60$

$x, y, s1, s2, s3 >= 0$

The Slack variables need to be introduced in the Objective function as well, so

Maximize $Z = 20x + 30y$

becomes:

Maximize $Z = 20x + 30y + 0s1 + 0s2 + 0s3$

The Slack Variables have 0 values in the Objective Function

Step 3 – Formulating a Basic Feasible Solution

- We have a basic feasible solution when we equate our variables to 0 i.e. $x = 0$ and $y = 0$

- Using Maximize $Z = 20x + 30y + 0s1 + 0s2 + 0s3$

  Subject to:

  $3x + 3y + s1 = 36$

  $5x + 2y + s2 = 50$

  $2x + 6y + s3 = 60$

  $x, y, s1, s2, s3 >= 0$

And setting $x = 0$ and $y = 0$, we get the values of s1, s2 and s3

Maximize $Z = 0$

$s1 = 36$

$s2 = 50$

$s3 = 60$

- Now, we make the First Simplex Tableau

Cj	Basic Variables	Solution/ Quantity	20	30	0	0	0	Ratio
			x	y	s1	s2	s3	
0	s1	36	3	3	1	0	0	36/3 = 12
0	s2	50	5	2	0	1	0	50/2 = 25
0	s3	60	2	6	0	0	1	60/6 = 10 ←
	Zj	36*0 + 50*0 + 60*0 = 0	3*0 + 5*0 + 2*0 = 0	3*0 + 2*0 + 6*0 = 0	1*0 + 0*0+0*0 = 0	0*0 + 1*0 + 0*0 = 0	0*0 + 0*0 + 1*0 = 0	
	Cj - Zj		20 – 0 =20	30 – 0 =30 ↑	0 – 0 = 0	0 – 0 = 0	0 – 0 = 0	

Step 4 – Test for Optimality

- Solution is Optimal if all values in the Cj-Zj row are either Negative or 0.
- If there is any positive value, then the solution can be improved.
- The variable with the highest Cj-Zj value will enter the table (the logic is that 30 > 20 i.e. it will bring in more profit, therefore, that variable should be given higher priority)

Step 5 – Deriving Key Row and Key Column

- Refer video for explanation

- The arrow marked ↑ indicates the *entering variable* i.e. y because it has the *highest Cj –Zj value*. It is called the **Key Column**
- The arrow marked ← indicates the *exiting variable* i.e. s3 because it has the *lowest ratio*. It is called the **Key Row.**
- The *intersection* of the Key Row and the Key Column is called the **Key Number** i.e. 6

- How to derive the "Ratio" Column

Each value in the "Solution/Quantity" column will be *divided by* each **positive** entry in the Key Column

The variable with the *minimum non-negative* value will depart/leave the table. This becomes the Key Row.

Step 6 – Deriving table 2 and testing that for optimality

- Since there is one variable that is departing, i.e. s3, and one variable entering, i.e. y, the values in each row will now have to be changed, to reflect changes in variables

i. Transformation of Key Row

   Divide each value of row s3 with the key number, i.e. 6.

   Replace the old values with the resulting values. The values of row "y" (entering variable) will be obtained.

ii. Transformation of Non-Key Rows

   New Non-Key Row = Old Non Key Row – (Key Column Entry x New Key Row Value)

Eg: For Row s1, we will calculate the new value under "Solution/Quantity" column as follows:

36 – 3 x 10 = 6

36 – Old Value

3 – The old value in the same row (s1) under the key column (y)

10 – The new key row number which we obtained in step (i) above i.e. 60/6 = 10

Now make Table/Tableau 2 (refer next slide)

Cj	Basic Variables	Solution/ Quantity	20	30	0	0	0	Ratio
			x	y	s1	s2	s3	
0	s1	6	2	0	1	0	-1/2	6/2 = 3 ←
0	s2	30	13/3	0	0	1	-1/3	30/(13/3) = 90/13
30	y	10	1/3	1	0	0	1/6	10/(1/3) = 30
	Zj	6*0 + 30*0 + 10*30 = 300	10	30	0	0	5	
	Cj - Zj		10 ↑	0	0	0	-5	

- Again, the solution is optimal when all values in the Cj-Zj row are either negative or 0
- In Simplex Tableau 2, we have a positive value in the "x" column, therefore, x becomes the entering variable
- All other values are either 0 or negative
- The lowest positive ratio is for row s1, therefore, s1 becomes the departing variable
- Using this, and transforming the table using formulae given in slide 13, we will derive simplex tableau 3 and test that for optimality

Cj	Basic Variables	Solution/ Quantity	20	30	0	0	0	Ratio
			x	y	s1	s2	s3	
20	x	**3**	1	0	1/2	0	-1/4	
0	s2	17	0	0	-13/6	1	3/4	
30	y	**9**	0	1	-1/6	0	1/4	
	Zj	3*20 + 17*0 + 9*30 = **330**	20	30	5	0	5/2	
	Cj-Zj		0	0	-5	0	-5/2	

Step 7 - Interpreting the Final Table and Deriving Solution

- Since all values in the C_j-Z_j row are either 0 or negative, we have found the optimum solution
- The answer is:

  Maximum Profit is 330

  When we produce:

  $x = 3$ i.e. 3 chairs &

  $y = 9$ i.e. 9 tables

  $s1, s3 = 0, s2 = 17$

- The C_j-Z_j row in the final tables shows the *shadow prices*

  i.e. for s1, it is 5, which means that if we increase the utilization of the resource by one additional hour, we can increase the value of our objective function (profit) by Rs.5
- The existence of a slack variable in the final table, i.e. $s2 = 17$, shows that our resources have not been completely utilized. At the same time, the value in the C_j-Z_j row for column $s2 = 0$, which again indicates that even if we add an additional hour for this resource, we will add Rs. 0 (or nothing) to the objective function.

- This topic was LPP for a Pure Maximization Case.
- Kindly practice: All solved examples from the book, starting from Ex 4 to Ex 12
- All unsolved questions asked in the past years